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Classical and quantum gravity of brane black holes

Ruth Gregory, Simon F. Ross and Robin Zegers

Centre for Particle Theory, Department of Mathematical Sciences,
Durham University, South Road, Durham DH1 3LE, U.K.
E-mail: r.a.w.gregory@durham.ac.uk, s.f.ross@durham.ac.uk, robin.zegers@durham.ac.uk

Abstract: We test the holographic conjecture of brane black holes: that a full classical 5D solution will correspond to a quantum corrected 4D black hole. Using the Schwarzschild-AdS black string, we compare the braneworld back reaction at strong coupling with the calculation of the quantum stress tensor on Schwarzschild-AdS$_4$ at weak coupling. The two calculations give different results and provide evidence that the stress tensor at strong coupling is indeed different to the weak coupling calculations, and hence does not conform to our notion of a quantum corrected black hole. We comment on the implications for an asymptotically flat black hole.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence.

Contents

1. Introduction

In exploring consequences of any quantum theory of gravity it is the nonperturbative questions that give the most fascinating opportunities for unexpected physical consequences. Black holes in particular have provided an extremely fruitful background for testing our understanding of quantum effects in gravity. While it has been known for some time that black holes emit Hawking radiation [1], the consequences of that radiation remain unproven. String theory has made huge advances in our understanding of black hole thermodynamics, but as yet is unable to access the highly non-supersymmetric Schwarzschild black hole. Clearly, any progress in understanding this physically relevant case would be extremely important.

Braneworlds are a framework in which the existence of large extra dimensions is allowed via a mechanism which confines standard model physics to a slice in spacetime, thus introducing potential hierarchies in interactions, as well as modifications of gravity at small (and sometimes large) scales. The brane typically warps the bulk spacetime, and in the Randall-Sundrum (RS) model [2, 3], is a slice through five dimensional anti-de Sitter spacetime. The RS model makes specific predictions for cosmology and the LHC dependent on the 5D AdS curvature scale. But the RS model has another interesting implication: by taking the near horizon limit of a stack of D3-branes, the RS model can be thought of as cutting off the spacetime outside the D-branes; the AdS curvature of the RS bulk is therefore given rather precisely in terms of the D3 brane charge and the string scale. Thus, from AdS/CFT [4], we might expect a parallel between classical branworld gravity, and quantum corrections on the brane.

There have been several attempts to utilize this relation, in the context of cosmology [5] and linearized gravity [6], for which the evidence is concrete and robust, and in the case of brane black holes [7, 8], for which the evidence is more circumspect, and open to criticism [9]. Briefly, a cosmological brane is a slice of a bulk black hole spacetime with the bulk black hole giving rise to a radiation source in the brane cosmology [10]. Comparing the temperature of this radiation to that of a field theory at finite Hawking temperature shows that these agree up to a factor [5]. For linearized gravity, the classical corrections to the RS brane can be computed from the Lichnerowicz operator, and there are specific $1/r^3$ corrections to the Newtonian potential [3]. These agree *precisely* with the 1-loop corrections to the graviton propagator in quantum gravity [6]. Given these results, it is tempting to suppose that a classical braneworld black hole solution will correspond to a quantum corrected black hole, however for this we need an actual solution!

The first attempt to find a braneworld black hole replaced the Minkowski metric in the RS model with a Schwarzschild metric, giving rise to an AdS black string [11]. This string however suffers from a classical instability [12], so it is not the correct bulk solution to describe a brane black hole. This instability might correspond to the thermodynamic instability of the Schwarzschild black hole via Hawking radiation, although the timescales and nature of the two instabilities seem to be rather different (see [13]).

This dual picture led to the conjecture that any nonsingular braneworld black hole solution must be time dependent [7, 8, 15]. However, the original argument for this relied on weak coupling calculations, whereas the bulk black hole solution corresponds to a strongly coupled field theory on the brane, so its behaviour may be very different. It was argued in [9] that the quantum-corrected dual description might be consistent with the existence of static localised black hole solutions. It is difficult to construct such solutions explicitly as the system of equations has too much freedom to be completely classified analytically [14], and the system is very numerically sensitive. So far, it has been possible to construct static nonsingular black hole solutions numerically, although these are for small masses $\lesssim O(\ell^{-1})$ [16].

Here we support the point of view of [9] by looking at a slightly modified RS brane - detuning the brane tension to subcritical, giving an anti-de Sitter, or Karch Randall (KR) [17] braneworld. We consider two KR branes which cross the AdS_5 bulk, both of positive tension, which intersect only formally on the AdS boundary. There are two types of bulk solution which correspond to a localised black hole from the braneworld point of view: a black string stretching between the two branes, or a bulk black hole which is localised near one brane and does not extend across the whole of the extra dimension. We focus on the black string, for which an explicit solution is known which is stable for a range of mass parameters. In the regime where it is stable, we would expect this black string to be the correct solution describing a brane black hole, and even when it is unstable, the bulk solution is regular, so it should have a boundary CFT description. We explore the description of this black string as a quantum-corrected black hole in the brane, and find that a consistent interpretation exists, but it involves surprising behaviour.

2. The Karch-Randall black string

We start by writing 5D AdS in a general form as a foliation over a 4-dimensional spacetime:

$$g = \Omega^2(u_{\mathrm{RS}})[du_{\mathrm{RS}}^2 + \tilde{g}], \tag{2.1}$$

where $\tilde{g}$ is a general 4-dimensional metric. The RS model takes $\tilde{g}$ to be Minkowski spacetime, with $\Omega_{\mathrm{RS}} = \ell/u_{\mathrm{RS}}$, and $\ell = \sqrt{-6/\Lambda}$ is the 5D AdS length. The AdS boundary is

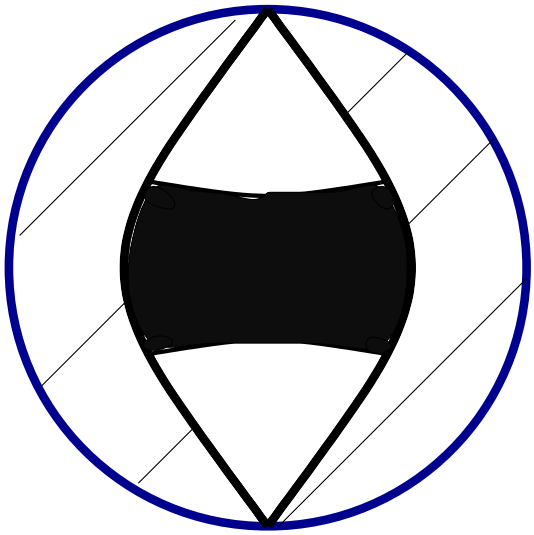

Figure 1: A sketch of the KR black string. The blue circle is the AdS boundary, which is excised from the braneworld spacetime. The string goes through the AdS bulk between the two KR branes. Because the string has finite proper length relative to its mass, it can be stable for sufficiently large mass.

at $u_{\rm RS} = 0$, and the RS brane is at constant $u_{\rm RS}$. To construct a KR brane, first make a simple change of coordinates: $u_{\rm RS} = r\cos\theta$. The AdS boundary now corresponds to $\theta = \pm\pi/2$, and

$$g = \frac{\ell^2}{\cos^2(\theta)}\left[d\theta^2 + \frac{\tilde{g}}{\tilde{\ell}^2}\right] \tag{2.2}$$

where $\tilde{g}$ is now an AdS_4 geometry with length scale $\tilde{\ell} = \ell\sec\theta_0$. Now introduce a brane at $\theta = \theta_0$. By Israel's equations, this brane has positive tension,

$$\sigma_{\rm KR} = \frac{6\sin\theta_0}{8\pi G_5 \ell} \tag{2.3}$$

which is less than the critical RS brane tension $\sigma_{\rm RS} = \frac{6}{8\pi G_5 \ell}$. This is a KR brane [17], and with a single brane is analogous to the one brane Randall-Sundrum model [3], however, unlike the RS model, we can include a second *positive tension* KR brane in the bulk at $\theta = -\theta_0$, which actually has the *same* tension as the first brane. Thus, unlike the original two brane RS model [2], in which the second brane had negative tension, and corresponded to a CFT cutoff in the UV and IR, the KR set-up has two positive tension branes, and thus corresponds to two CFT's cut off in the UV. The distance between these two branes is *finite*:

$$D = \int_{-\theta_0}^{\theta_0} \ell\sec\theta d\theta = 2\ell\ln\left(\frac{\tilde{\ell}}{\ell} + \frac{4\pi G_5 \sigma_{\rm KR}\tilde{\ell}}{6}\right) . \tag{2.4}$$

Thus the graviton spectrum consists of a zero mode of 4D gravity, a radion, and a discrete tower of KK states, with masses given by the appropriate brane boundary conditions.

Now let $\tilde{g}$ be the metric of a Schwarzschild-AdS$_4$ black hole [18]:

$$\tilde{g} = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2.5}$$

with

$$V(r) = 1 + \frac{r^2}{\tilde{\ell}^2} - \frac{2G_4 M}{r}. \tag{2.6}$$

This has a horizon at r_+ satisfying $V(r_+) = 0$. This black string stretches between the two branes (see figure 1), and analogous to the RS black string, we expect that it will exhibit an instability. This system was analysed in the absence of branes by [19], who found stability for $r_+ \gtrsim O(\tilde{\ell})$. There are some technical issues with divergence of transverse eigenfunctions in their analysis, however, we have checked that their conclusion is correct in the presence of the branes. Thus, for large $r_+/\tilde{\ell}$, the black string is a suitable brane plus bulk solution for a KR braneworld black hole. For small $r_+/\tilde{\ell}$ (and in particular, in the limit as the brane cosmological constant goes to zero), this solution is unstable, and should decay into a localised solution, which would not be of the simple warped product form we have considered.[1]

3. Corrections to the braneworld black hole

We therefore have (for large mass) a stable classical bulk solution which from the point of view of the brane is exactly a Schwarzschild-AdS$_4$ black hole. We would like to understand how this is reconciled with the viewpoint of [7, 8], that the bulk geometry describes, from the dual brane/CFT point of view, a quantum-corrected black hole. One would have expected in general that the back-reaction of the quantum stress tensor would change the form of the geometry. There is a non-zero $\mathcal{O}(N^2)$ stress tensor for the $\mathcal{N} = 4$ SYM theory on this background, as can be seen by considering the conformal anomaly, which is

$$\langle T^\mu{}_\mu \rangle = \frac{N^2 - 1}{32\pi^2} \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right). \tag{3.1}$$

On an Einstein space-time such as Schwarzschild-AdS$_4$, we thus have

$$\langle T^\mu{}_\mu \rangle = -\frac{N^2 - 1}{24\pi^2} \tilde{\Lambda}^2 = -\frac{3(N^2 - 1)}{8\pi^2 \tilde{\ell}^4}. \tag{3.2}$$

So the stress tensor should produce a back-reaction whose effects would be visible at the order we are considering. Why do we see simply a Schwarzschild-AdS$_4$ geometry?

The solution is that, as in the discussion of the pure Schwarzschild black string in [9], the form of the stress tensor we predict for the strongly-coupled CFT dual to the bulk

[1] Note, in [20], a different instability for foliations of AdS in terms of compact negatively curved spaces was discussed. From the braneworld point of view, this instability corresponds to a scalar tachyonic mode. This mode is also present in the solutions we consider here; however, it satisfies the Breitenlohner-Freedman bound for the brane spacetime, so in the present context, where we are considering a non-compact braneworld spacetime, it does not imply an instability so long as we impose the usual asymptotically AdS$_4$ boundary conditions on the braneworld spacetime.

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geometry is very special. We can evaluate the full stress tensor for the boundary field theory by using the bulk spacetime and applying the boundary stress tensor/holographic renormalization approach of [21, 22]. In this approach, we expand the bulk metric as

$$g = \frac{dz^2}{z^2} + \frac{1}{z^2}\left[\tilde{g}_{(0)} + z^2\tilde{g}_{(2)} + z^4\tilde{g}_{(4)} + \ldots\right], \tag{3.3}$$

and then the stress tensor can be evaluated as [22]

$$\begin{aligned}\langle T_{\mu\nu}\rangle = \frac{\ell^3}{4\pi G_5}\Bigg[&\tilde{g}_{(4)\mu\nu} + \frac{1}{8}\left(\mathrm{tr}(\tilde{g}^2_{(2)}) - \left(\mathrm{tr}\,\tilde{g}_{(2)}\right)^2\right)\tilde{g}_{(0)\mu\nu} \\ &-\frac{1}{2}\left(\tilde{g}^2_{(2)}\right)_{\mu\nu} + \frac{1}{4}\tilde{g}_{(2)\mu\nu}\mathrm{tr}\,\tilde{g}_{(2)}\Bigg]\end{aligned} \tag{3.4}$$

(where we have ignored some logarithmic terms in the generic expression which will not contribute in our case). In our case, the metric (2.2) can be brought into the appropriate form by writing

$$\sec\theta = \frac{4\tilde{\ell}^2 + z^2}{4\tilde{\ell}z} \tag{3.5}$$

so that

$$g = \frac{\ell^2}{z^2}\left[dz^2 + \left(1 + \frac{z^2}{2\tilde{\ell}^2} + \frac{z^4}{16\tilde{\ell}^4}\right)\tilde{g}\right]. \tag{3.6}$$

Thus we find

$$\langle T_{\mu\nu}\rangle = -\frac{3\ell^3}{64\pi G_5\tilde{\ell}^4}\tilde{g}_{\mu\nu} = -\frac{3N^2\hbar}{32\pi^2\tilde{\ell}^4}\tilde{g}_{\mu\nu}, \tag{3.7}$$

where we use $\hbar G_5 = \frac{\hbar G_{10}}{\pi^3\ell^5} = \frac{\pi\ell^3}{2N^2}$ in the last step. The key point is that this stress tensor is proportional to the metric on the boundary; the effects of the back-reaction will therefore be solely to renormalize the four-dimensional cosmological constant. This special form for the stress tensor arises directly from the foliated form of the five-dimensional metric. This is also consistent with the arguments of [9] for the case $\tilde{\Lambda} = 0$: as $\tilde{\ell} \to \infty$, $\langle T_{\mu\nu}\rangle \to 0$, so this leading $\mathcal{O}(N^2)$ part of the quantum stress tensor vanishes in this limit.

It is interesting to compare this with the fully nonlinear classical result obtained at arbitrary cut-off using the Israel formalism. Here, the brane and bulk metric are completely specified, and the correction to the brane energy momentum is interpreted via the discrepancy between this solution and the conventional 4D Einstein equation.

Note that we can interpret the KR brane as the critical RS brane with a stress tensor source on the brane corresponding to a negative "cosmological constant" λ:

$$\frac{6\sin\theta_0}{8\pi G_5\ell} = \frac{6}{8\pi G_5\ell} + \lambda \tag{3.8}$$

On the other hand, the *actual* 4D cosmological constant is given by

$$\Lambda_4 = -\frac{3}{\tilde{\ell}^2} = 8\pi G_4\lambda_{\text{eff}}\,. \tag{3.9}$$

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Note that in this case, the 4D gravitational constant is not labelled as G_N, since the relation between the brane and bulk gravitational constant is dependent on the brane tension, and is not given by the standard RS relation in terms of ℓ [23]:

$$G_4 = \frac{4\pi G_5 \sigma_{\rm KR}}{3} G_5 \,. \tag{3.10}$$

From the definition of $\tilde{\ell}$ and (3.8), (3.10), the value of the bare tension is:

$$\lambda = \frac{3}{4\pi G_4 \ell^2 \tilde{\ell}^2} \left(\tilde{\ell}^2 - \ell^2 - \tilde{\ell}\sqrt{\tilde{\ell}^2 - \ell^2} \right) \tag{3.11}$$

Therefore, since the 'expected' value of the cosmological constant is $8\pi G_4 \lambda$, we can compute the correction to the brane energy momentum as:

$$\langle T^\mu_\nu \rangle = \frac{8\pi G_4 \lambda - 3/\tilde{\ell}^2}{8\pi G_4} \delta^\mu_\nu = \frac{3(2\tilde{\ell}^2 - \ell^2 - 2\tilde{\ell}\sqrt{\tilde{\ell}^2 - \ell^2})}{8\pi G_5 \ell \tilde{\ell} \sqrt{\tilde{\ell}^2 - \ell^2}} \delta^\mu_\nu \tag{3.12}$$

This is the precise (classical) braneworld result. We can obtain the holographic renormalization result (3.7), by taking the limit as the brane approaches the boundary, or by approaching the critical RS limit $\lambda \to 0$, $\tilde{\ell} \to \infty$:

$$\langle T^\mu_\nu \rangle = \frac{3\ell^3}{32\pi G_5 \tilde{\ell}^4} \delta^\mu_\nu, \tag{3.13}$$

which agrees with (3.7) up to the expected factor of two which arises from the braneworld set-up having two copies of the bulk, one on each side of the brane.

Thus, the bulk solution can be consistently interpreted as a quantum-corrected metric in the dual boundary theory. However, the form of the boundary stress tensor obtained by this argument is very different from what we would expect. Our result is independent of the black hole temperature, whereas we would have expected a component corresponding to a thermal plasma of CFT degrees of freedom outside the black hole. The form of a thermal plasma in the strong coupling CFT is known from AdS/CFT [24]. No such contribution can be seen in (3.7).

To see the contrast with the expected behaviour in detail, is instructive to compare the above holographic calculation to a weak-coupling calculation of the stress tensor of a quantum field on Schwarzschild-AdS$_4$. We will consider a conformally coupled scalar field, where an approximate calculation of the quantum stress tensor on Einstein spaces by Page [25] can be applied. In Page's approach, we analytically continue to Euclidean signature and consider the conformally related optical metric, $g_{\rm opt} = \Omega^{-2}\tilde{g}$ with $\Omega = V(r)^{-1/2}$, where $V(r)$ is given in (2.6). In the Euclidean space, to ensure smoothness at the horizon, τ is periodically identified with period $\tau \sim \tau + 1/T$, where the temperature $T = V'(r_+)/4\pi = (\tilde{\ell}^2 + 3r_+^2)(4\pi\tilde{\ell}^2)$. We also write the mass appearing in (2.6) in terms of r_+ as $G_4 M = r_+ \left(1 + r_+^2 \tilde{\ell}^{-2}\right)/2$. Page shows that in a Gaussian approximation to the heat kernel [26], the stress tensor of the scalar field in this optical metric can be approximated by $\langle T^\mu{}_\nu \rangle_{\rm opt} = \frac{\pi^2}{90} T^4 (\delta^\mu{}_\nu - 4\delta^\mu{}_0 \delta^0{}_\nu)$. The stress tensor in the physical metric can then be determined using the properties of the field under a conformal transformation.

Applying this to the Schwarzschild-AdS$_4$ geometry, we find

$$\langle T^\mu{}_\nu \rangle = \frac{\pi^2}{90(4\pi r_+)^4} \frac{1}{r^6} \left[T^{(1)}(r) \left(\delta^\mu{}_\nu - 4\delta^\mu{}_0 \delta^0{}_\nu \right) + 3T^{(2)}(r) \delta^\mu{}_0 \delta^0{}_\nu + T^{(3)}(r) \delta^\mu{}_1 \delta^1{}_\nu \right] , \tag{3.14}$$

where we have set

$$\begin{aligned} T^{(1)}(r) =& \left(\frac{r - r_+}{rV(r)} \right)^2 \Bigg[\left(r^2 + 2rr_+ + 3r_+^2 \right) \left(r^4 + 4rr_+^3 - 3r_+^4 \right) \\ &+ \frac{4r_+^2}{\tilde{\ell}^2} \left(3r^6 + 6r^5 r_+ + 9r^4 r_+^2 + 8r^3 r_+^3 + r^2 r_+^4 - 9r_+^6 \right) \\ &+ \frac{2r_+^4}{\tilde{\ell}^4} \left(7r^6 + 38r^5 r_+ + 33r^4 r_+^2 + 20r^3 r_+^3 - 17r^2 r_+^4 - 18rr_+^5 - 27r_+^6 \right) \\ &- \frac{12r_+^4}{\tilde{\ell}^6} \left(4r^8 + 8r^7 r_+ + 3r^6 r_+^2 - 6r^5 r_+^3 - 3r^4 r_+^4 + 5r^2 r_+^6 + 4rr_+^7 + 3r_+^8 \right) \\ &- \frac{3r_+^4}{\tilde{\ell}^8} \left(8r^{10} + 16r^9 r_+ + 24r^8 r_+^2 + 32r^7 r_+^3 + 13r^6 r_+^4 - 6r^5 r_+^5 \right. \\ &\left. - r^4 r_+^6 + 4r^3 r_+^7 + 9r^2 r_+^8 + 6rr_+^9 + 3r_+^{10} \right) \Bigg] \end{aligned} \tag{3.15}$$

$$T^{(2)}(r) = 8r_+^4 \left(3r_+^2 + \frac{6r_+^4}{\tilde{\ell}^2} - \frac{2r_+ r^3}{\tilde{\ell}^2} + \frac{3r_+^6}{\tilde{\ell}^4} - \frac{2r_+^3 r^3}{\tilde{\ell}^4} - \frac{4r^6}{\tilde{\ell}^4} \right) \tag{3.16}$$

$$T^{(3)}(r) = 24r_+^5 \left(1 + \frac{r_+^2}{\tilde{\ell}^2} \right) \left(r_+ + \frac{r_+^3}{\tilde{\ell}^2} + \frac{2r^3}{\tilde{\ell}^2} \right) \tag{3.17}$$

This exhibits the expected thermal behaviour. A useful check of the analysis is to note that $\langle T_{\mu\nu} \rangle$, as given by (3.14), is regular at the horizon $r = r_+$, as $\langle T^0{}_0(r_+) \rangle = \langle T^1{}_1(r_+) \rangle$. We can also see that in the regime where $r, r_+ \ll \tilde{\ell}$, we recover Page's result [25] for the asymptotically flat Schwarzschild black hole.

This weak coupling result can also be used to consider the behaviour for large black holes, which was recently considered in [27]. In the regime where $r_+ \gg \tilde{\ell}$, let us write $r = zr_+$. In terms of z, (3.14) reads, at leading order,

$$\begin{aligned} \left\langle \bar{T}^\mu{}_\nu(z) \right\rangle =& \frac{1}{5760\pi^2} \frac{1}{\tilde{\ell}^4 z^6} \Bigg[\frac{-3F(z)}{(1 + z + z^2)^2} \left(\delta^\mu{}_\nu - 4\delta^\mu{}_0 \delta^0{}_\nu \right) + 24 \left(3 - 2z^3 - 4z^6 \right) \delta^\mu{}_0 \delta^0{}_\nu \\ &+ 24 \left(1 + 2z^3 \right) \delta^\mu{}_1 \delta^1{}_\nu \Bigg] + \mathcal{O}\left(\frac{1}{r_+^2 \tilde{\ell}^2} \right) , \end{aligned} \tag{3.18}$$

where F is a polynomial of order 10, $F(z) = 8z^{10} + \cdots + 3$. Thus, we see that at large r_+, the quantum stress tensor does not become large; the factors of r_+ cancel out. This shows directly that quantum corrections remain under control in this regime, as was argued by other methods in [27].

4. Discussion

To sum up: We see that the bulk solution can formally be interpreted as a quantum-corrected black hole on the brane, but the stress tensor involved does not have the expected form: it has the peculiar feature that the correction (a renormalization of the cosmological constant) is independent of the black hole mass. We also computed the weak coupling stress tensor, which has the expected form.

It is the strong coupling effect that is particularly intriguing: why should the black hole apparently not radiate at all in our solution? This does not seem to be physically sensible. This special form at strong coupling is a direct consequence of the fact that the bulk spacetime is foliated by conformal copies of the Schwarzschild-AdS black hole. This 'translation invariance' means that the classical KK graviton modes are not excited in the background solution, and geometrically the only possibility is renormalization of the cosmological constant. This is very different from what we would generically expect at strong coupling, and appears to indicate that the black hole does not have any thermal radiation associated with it. It also differs from the weak coupling stress-energy tensor, which has a more complicated form consistent with our physical picture of a radiating black hole in an AdS 'box'.

One might therefore ask if we are considering the correct solution. After all, black hole solutions are known not to be unique in 5D, and it is possible that there is a solution which is highly nonuniform in the bulk, which nonetheless has the form of a black hole on its intersection with the brane. The bulk horizon might have the pancake structure of the Randall Sundrum brane black hole conjectured by [11], or, given that the distance between the KR branes is finite, it might be a nonuniform horizon between the branes. However, in either case we have to compare the putative nonuniform solution with the known, stable static configuration we have used for $r_+ > \tilde{\ell}$. Not only is it difficult to envisage how a stable bulk horizon could intersect one brane without impinging on the other, but work on the related set-up of KK black holes in the 5D vacuum [28] indicates that in the equivalent mass régime, the black string solution is entropically preferred. We therefore believe we have chosen a good solution.

One possible open issue is whether one should consider a solution not of Einstein gravity, but of Einstein-Gauss-Bonnet theory. This would introduce inhomogeneities in the bulk direction, so that even the black string solution would not be of the simple foliated form. One could then ask if it still exhibited such peculiar features in the dual description. The main objection to this is that higher order corrections to the Einstein action occur at $\mathcal{O}(\alpha')$ in the string action, and our strong coupling computation is in the $N \to \infty, \alpha' \to 0$ limit, in which such corrections should have no effect.

Our calculation shows again that there are real questions about the interpretation of classical bulk solutions as quantum corrected brane solutions. Sufficient puzzles remain that this will no doubt continue to be a source of lively debate.

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